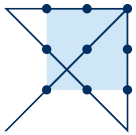


HITS
Heidelberg Institute for
Theoretical Studies

Probabilistic Flux Variation Gradient

Nikos Gianniotis, Francisco Pozo Nunes, Kai Polsterer
Astroinformatics Group
https://www.h-its.org/research/ain/

Overview: the big puzzle

- The ultimate goal is to (re)estimate the Hubble constant.
  This sheds light on the evolution of the universe.

- A crucial element is the estimation of black hole masses in AGN.
  We use AGN lightcurves which are however contaminated by the host-galaxy.

- Galaxy contribution has been estimated via the Flux Variation Gradient (FVG).
  But FVG does not account observation noise, neither quantifies uncertainty of its estimate.

- <u>This work:</u> formulate a probabilistic FVG that overcomes these issues.

This talk: our puzzle piece

- Begin with a brief overview of the Flux Variation Gradient (FVG)
  FVG understood as intersection of two lines

- Identify shortcomings: account for noise and uncertainty

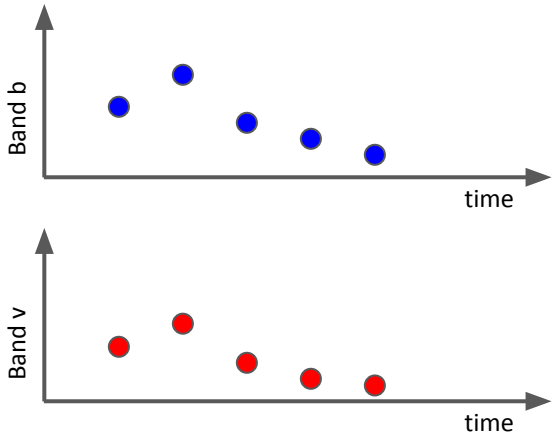
- Formulate a probabilistic version of FVG (PFVG)
  Intersection of a line with a distribution of lines

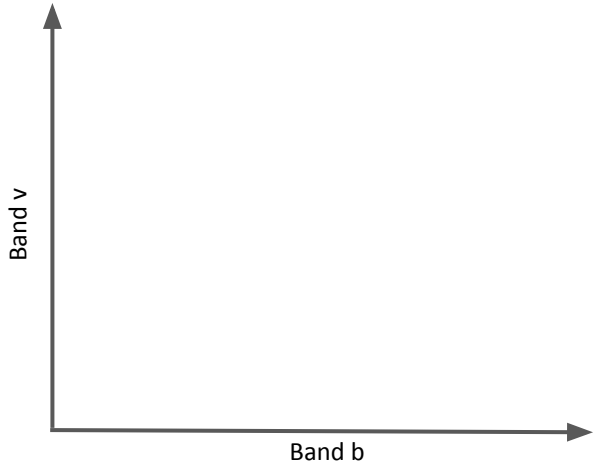
- Example application on 3C120

Flux variation gradient

Assumptions

1. Total fluxes (AGN+galaxy) at different bands follow linear relationship (empirically known)
2. Know direction of unknown, constant galaxy vector
3. AGN fluxes form a line that goes through the origin





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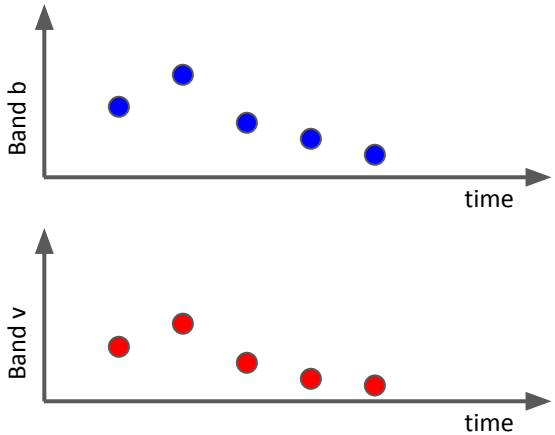

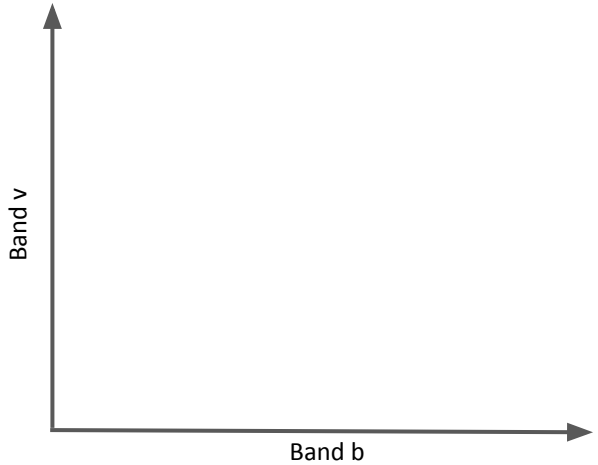

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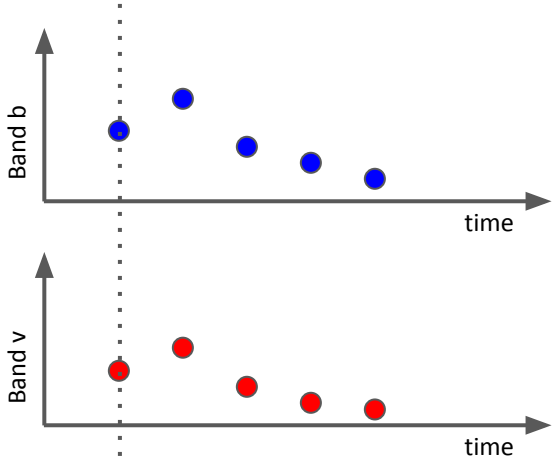

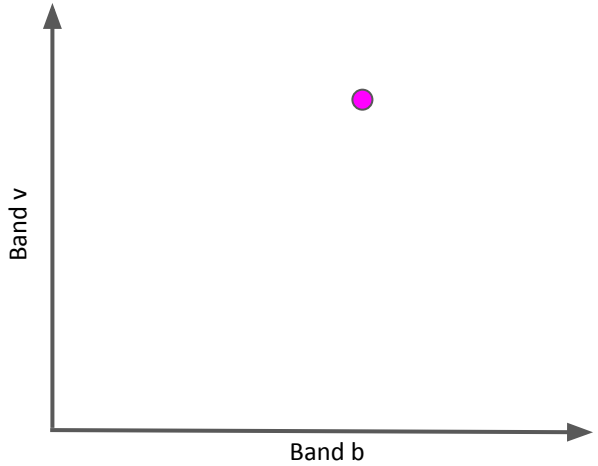

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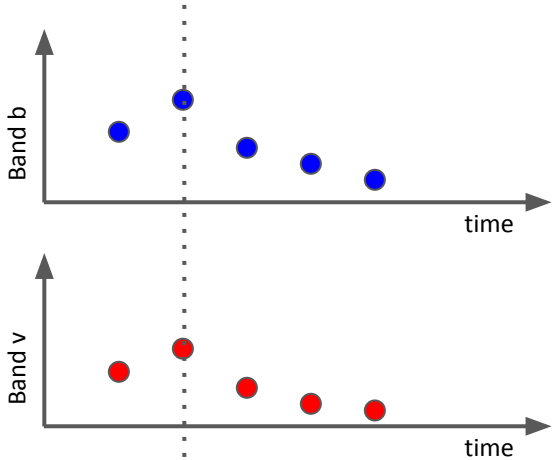

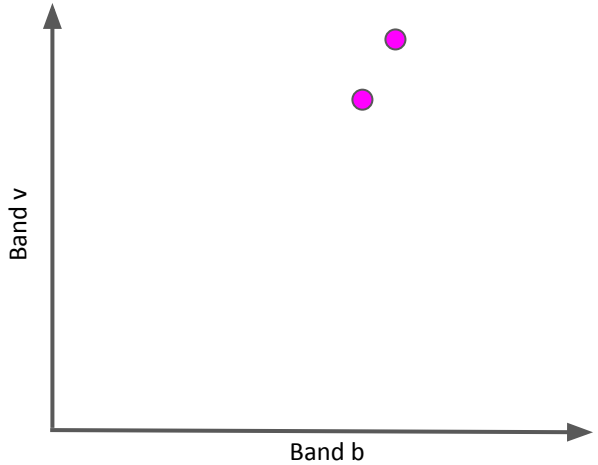

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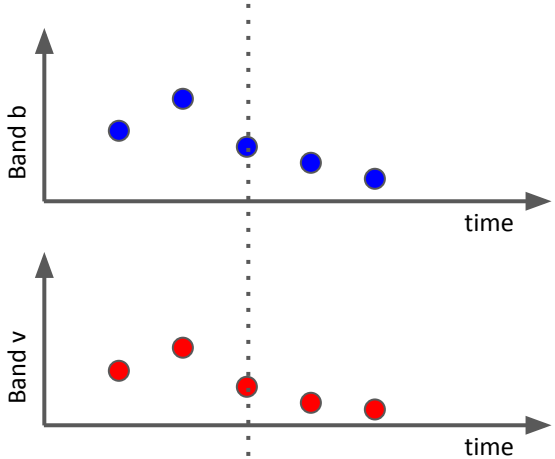

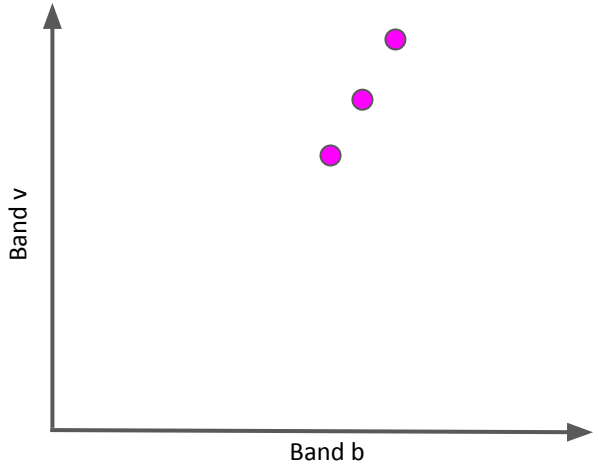

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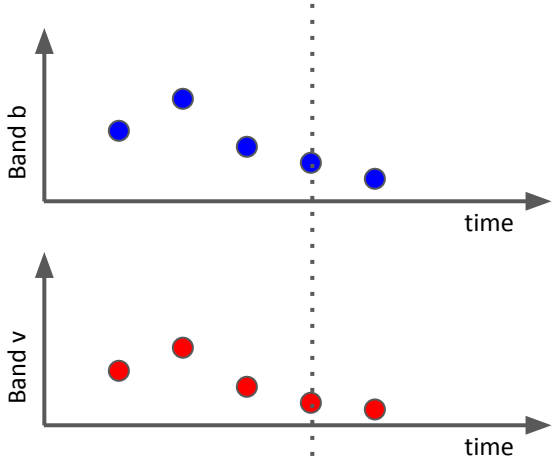

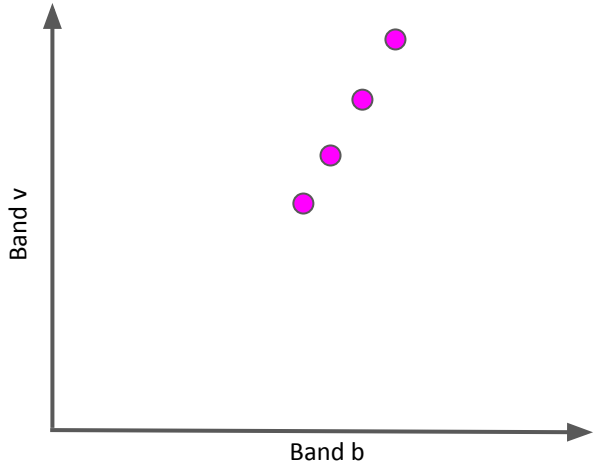

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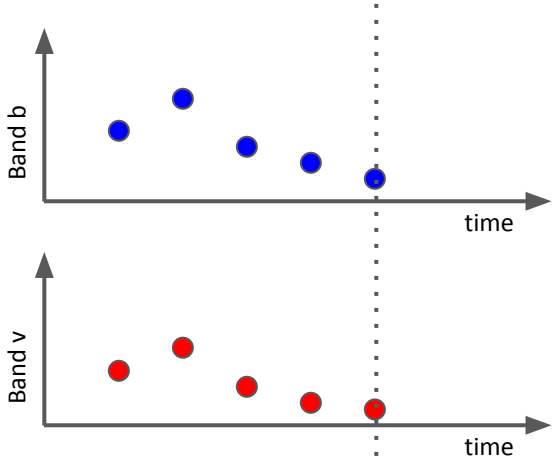

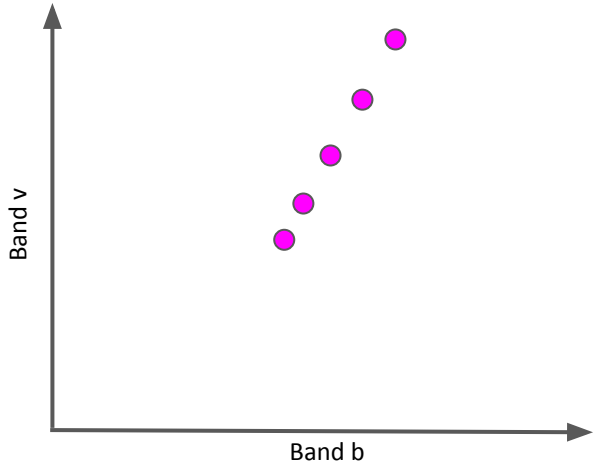

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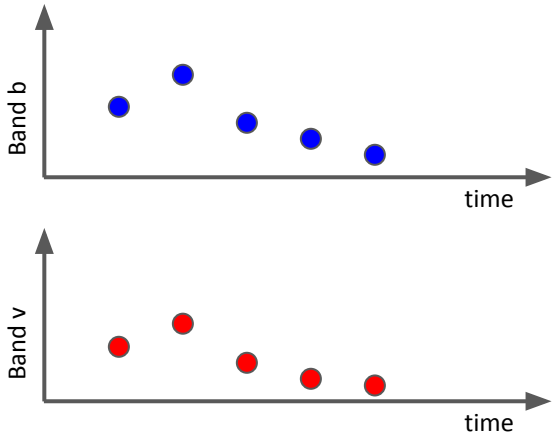

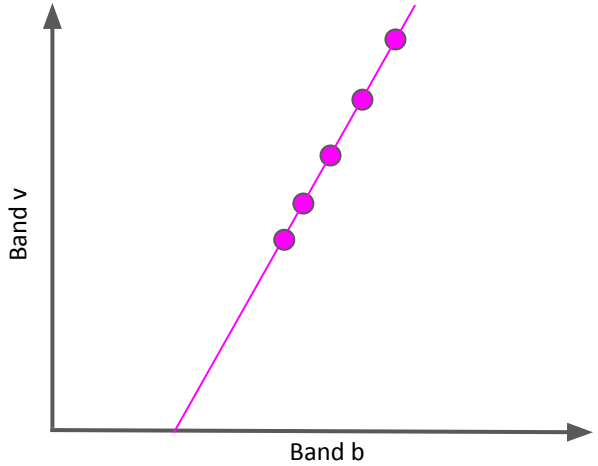

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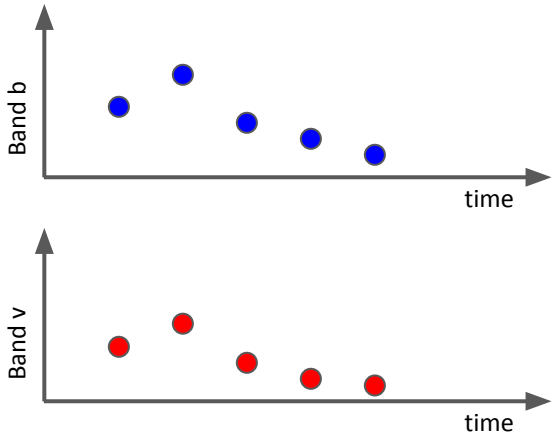

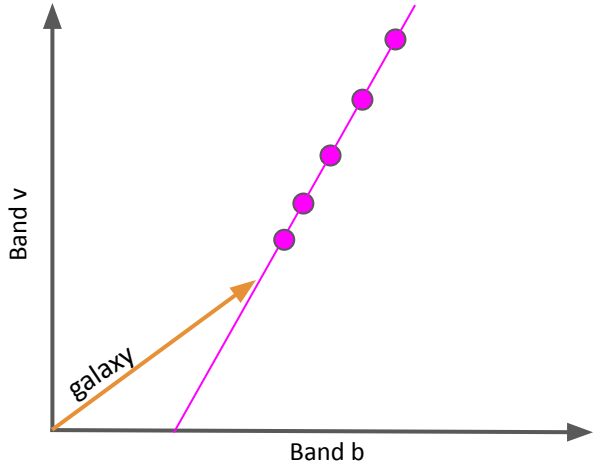

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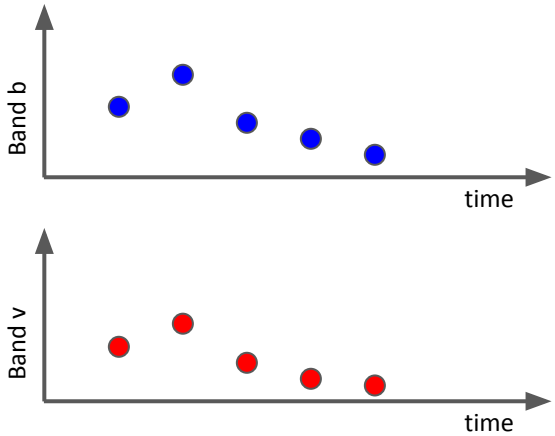

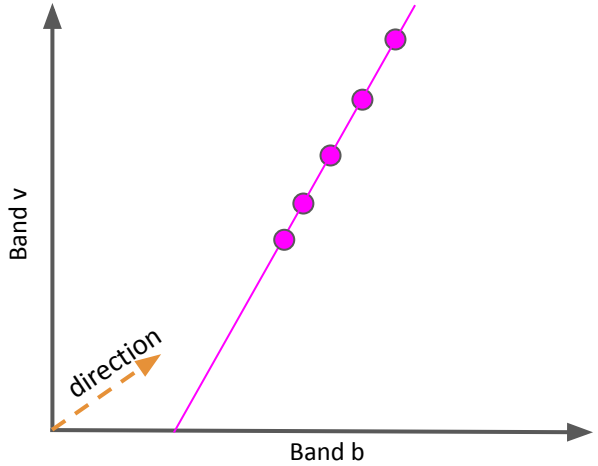

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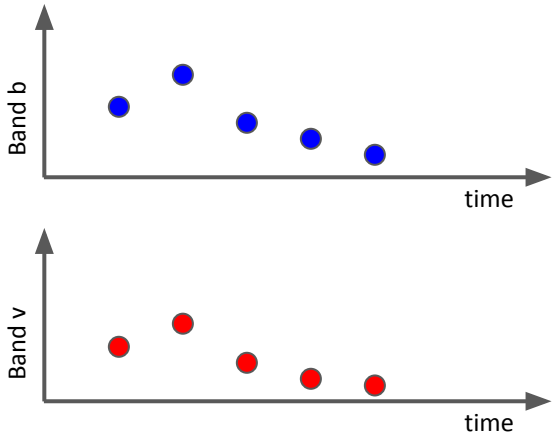

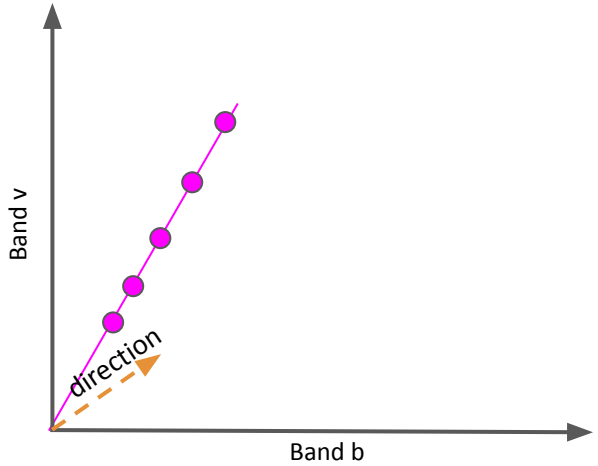

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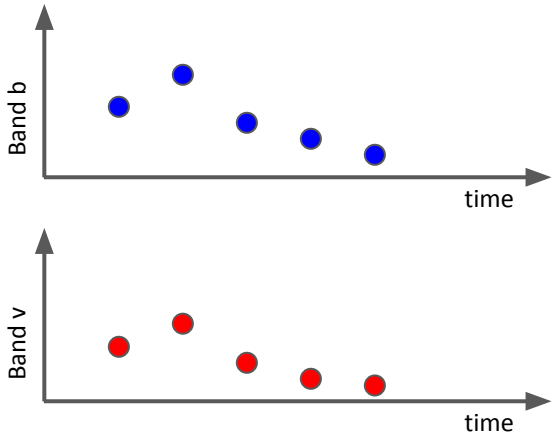

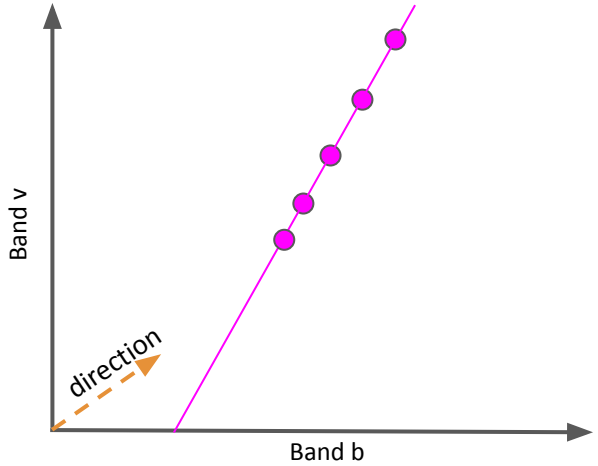

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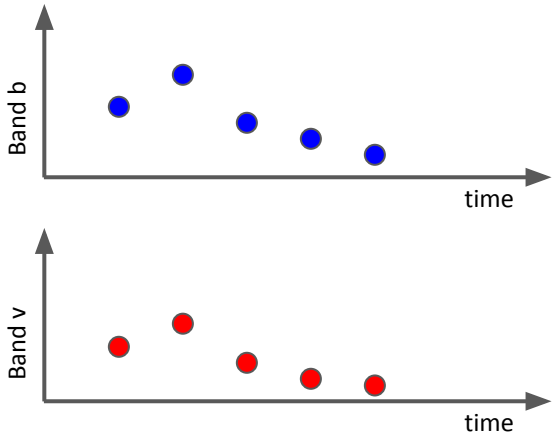

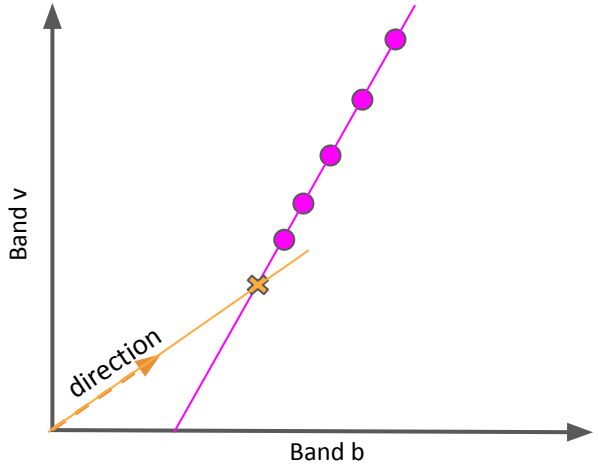

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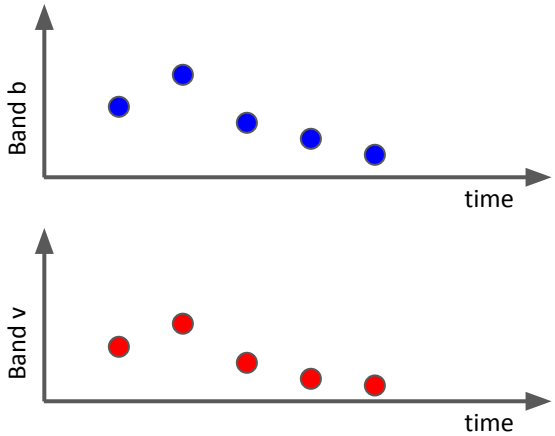
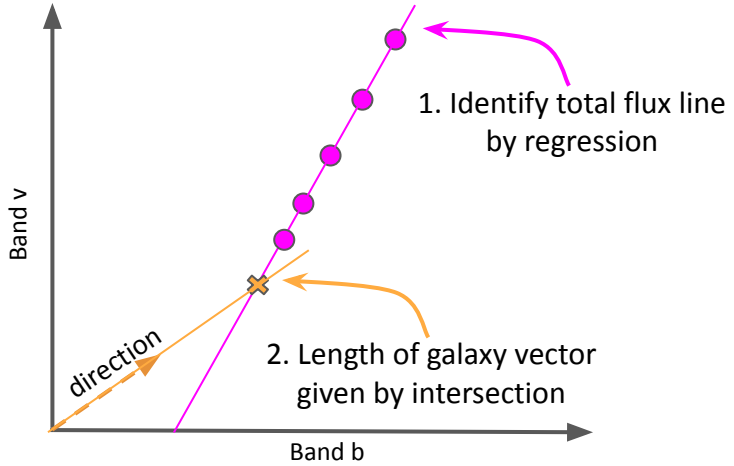
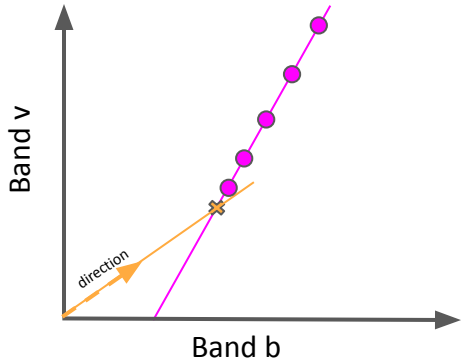




Flux variation gradient

Disadvantages

1. Identifying line by regression depends on which band is input, what band is output
2. Not accounting for observation noise, no uncertainty quantification
3. Heuristic aggregation of flux-flux plots



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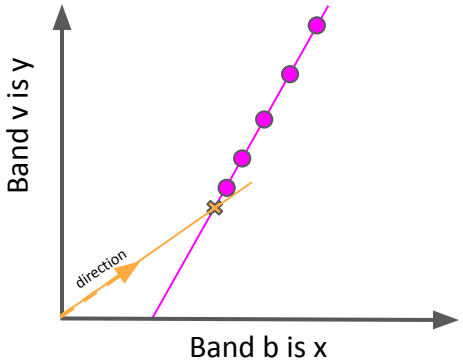

$$w = (x^T x)^{-1} x^T y$$

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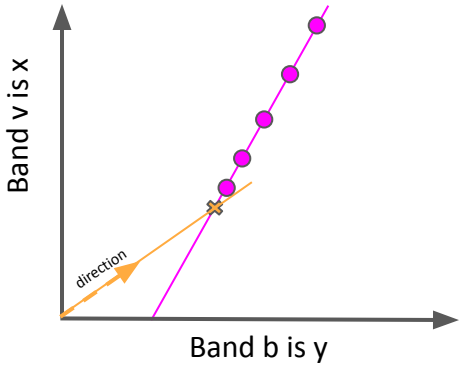

$$w = (x^T x)^{-1} x^T y$$

$$w = (y^T y)^{-1} y^T x$$

For each solution w, we find
a findumentally different intersection.
Not due to switching axes!

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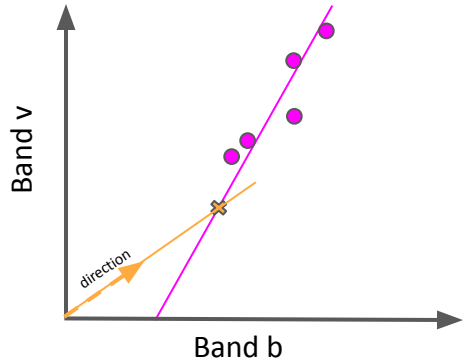

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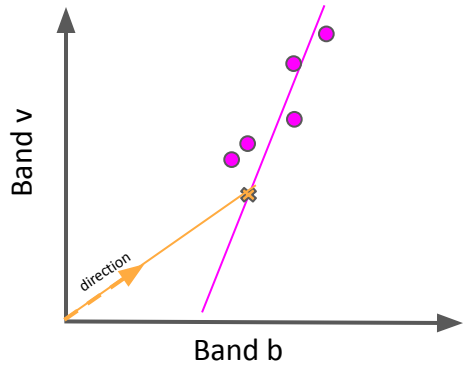

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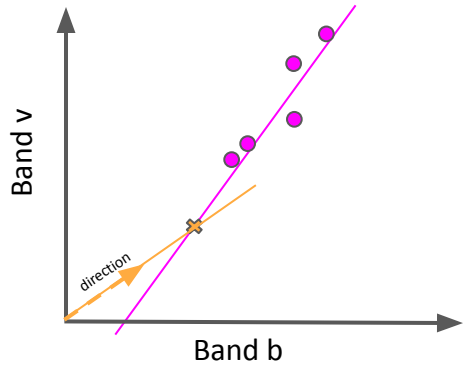

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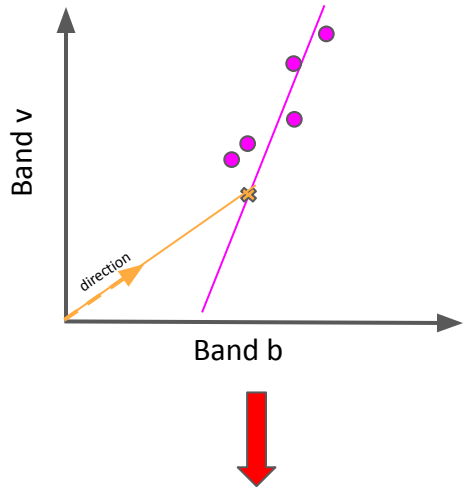
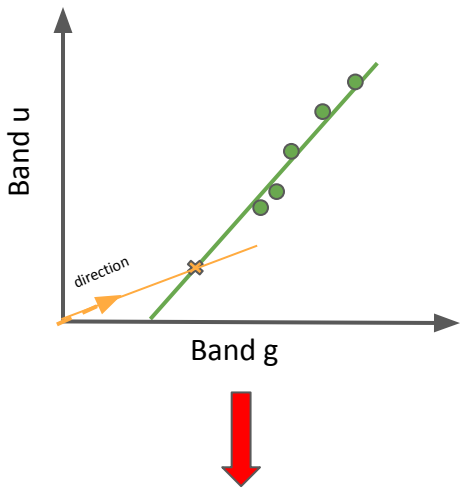
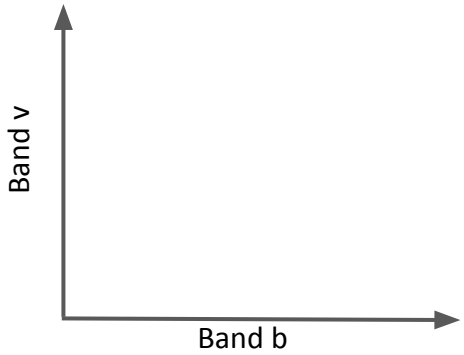


Length of galaxy vector is ℓ



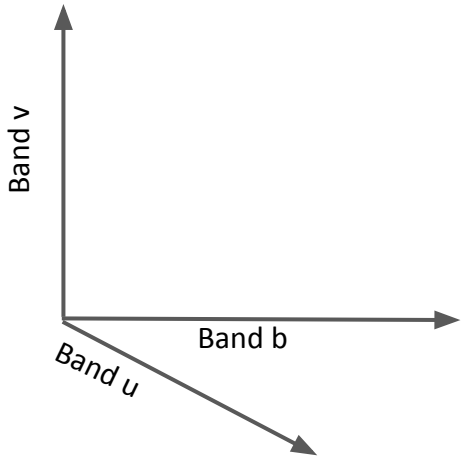
Length of galaxy vector is ℓ'

Probabilistic flux variation gradient - line identification



1. Instead of working with multiple flux-flux plots, we work with a single multidimensional flux plot

Probabilistic flux variation gradient - line identification



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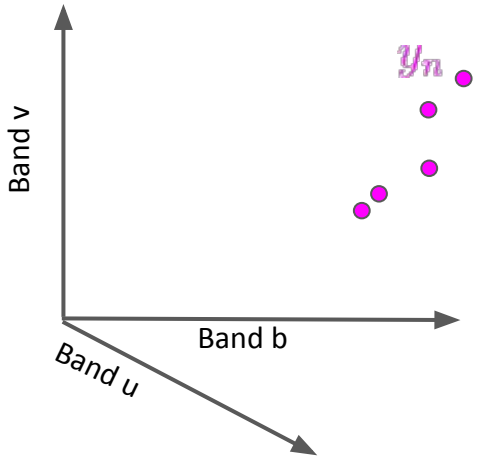

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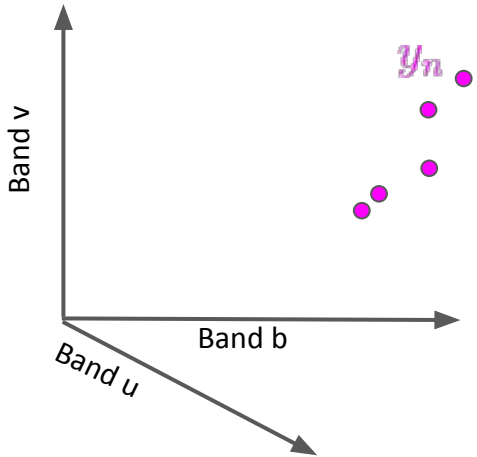

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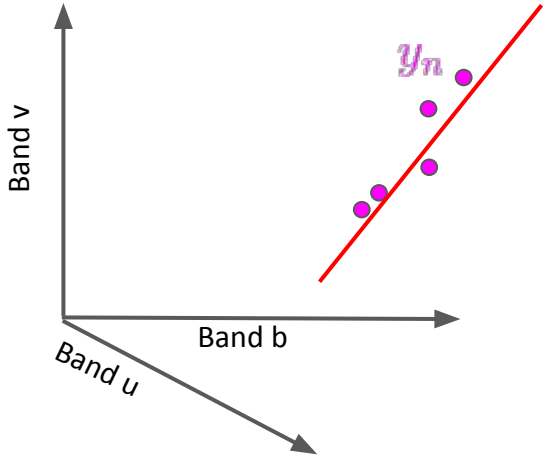

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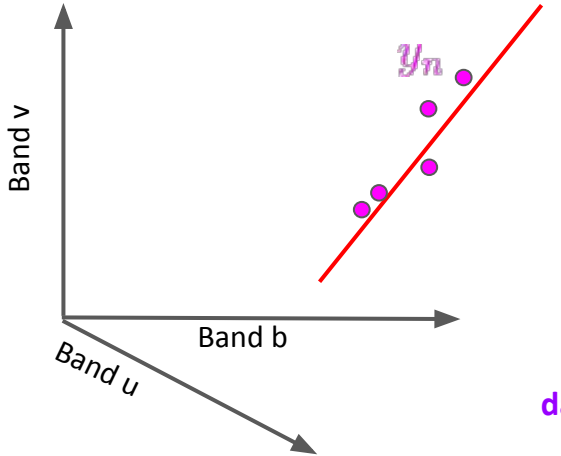
$$\prod_{n=1}^{N} \mathcal{N}(\mathbf{y}_n|\boldsymbol{\mu}, \mathbf{W}\mathbf{W}^T + \sigma_n^2\mathbf{I})$$

Probabilistic flux variation gradient - line identification



data mean

principal eigenvector(s)

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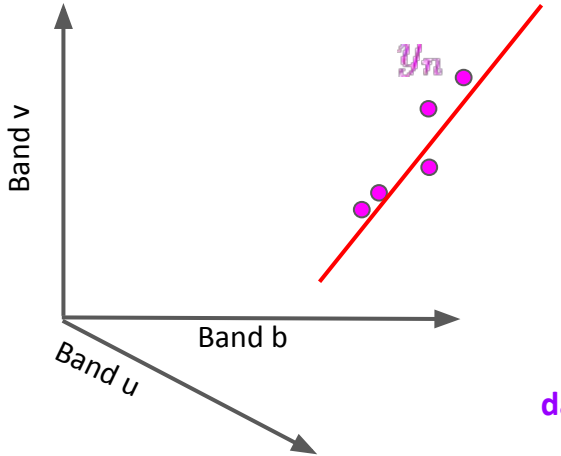

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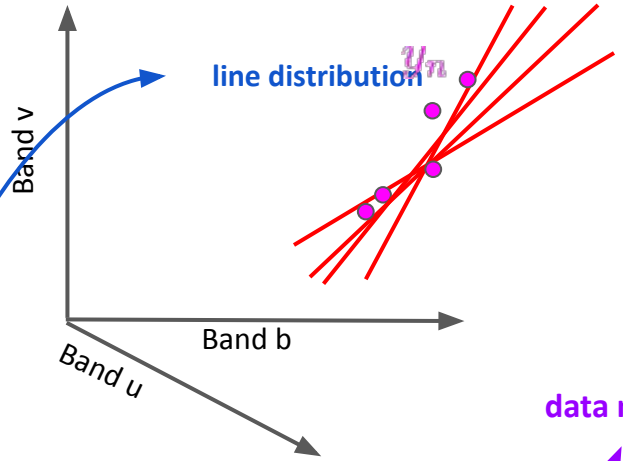
3. Instead of identifying a single line, we identify a distribution of lines

$$p(\mathbf{W}|\mathcal{D}) \propto \int \prod_{n=1}^{N} \mathcal{N}(\mathbf{y}_n|\boldsymbol{\mu}, \mathbf{W}\mathbf{W}^T + \sigma_n^2\mathbf{I})\, \mathcal{N}(\mathbf{W}|\mathbf{0}, \alpha^{-1}\mathbf{I})$$

Probabilistic flux variation gradient - line identification



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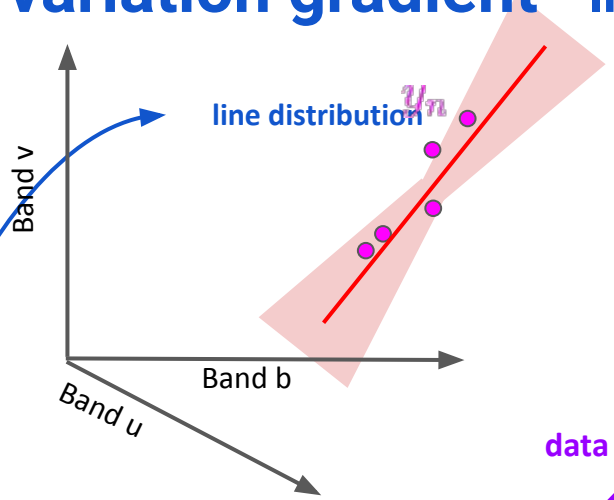
$$p(\mathbf{W}|\mathcal{D}) \propto \int \prod_{n=1}^{N} \mathcal{N}(\mathbf{y}_n|\boldsymbol{\mu}, \mathbf{W}\mathbf{W}^T + \sigma_n^2\mathbf{I})\, \mathcal{N}(\mathbf{W}|\mathbf{0}, \alpha^{-1}\mathbf{I})$$

Probabilistic flux variation gradient - line identification



data mean

principal eigenvector(s)

1. Instead of working with multiple flux-flux plots, we work with a single multidimensional flux plot
2. Instead of using regression, we identify the total flux line with probabilistic PCA

$$\prod_{n=1}^{N} \mathcal{N}(\mathbf{y}_n|\boldsymbol{\mu}, \mathbf{W}\mathbf{W}^T + \sigma_n^2\mathbf{I})$$

3. Instead of identifying a single line, we identify a distribution of lines

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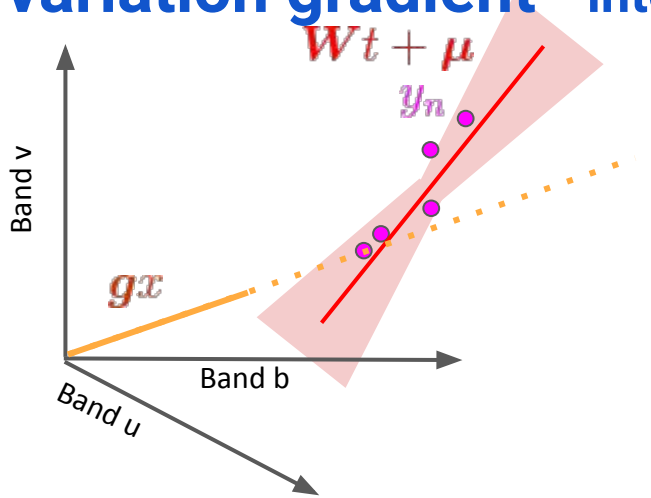
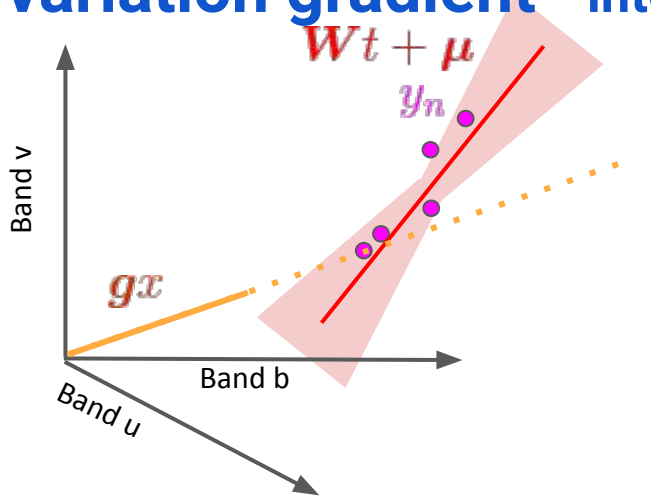
Probabilistic flux variation gradient - intersection



What does it mean to intersect a distribution of lines?

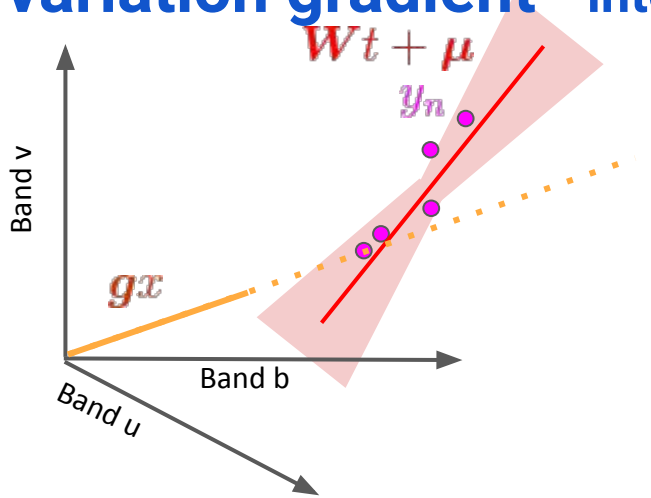
Probabilistic flux variation gradient - intersection



What does it mean to intersect a distribution of lines?

gx $\quad\quad\quad\quad$ $Wt + \mu$

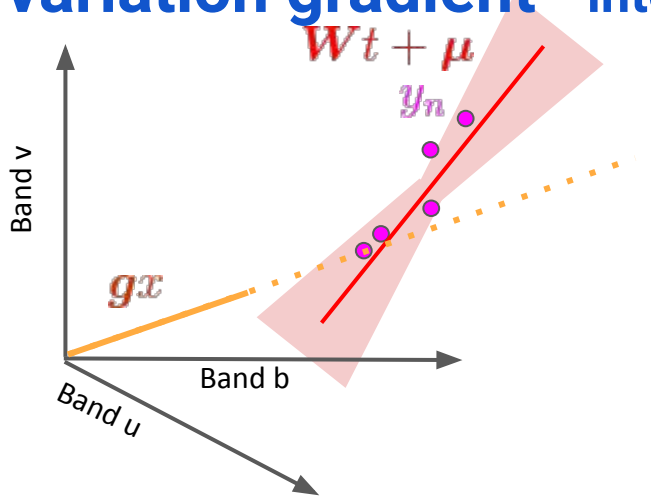
Probabilistic flux variation gradient - intersection



What does it mean to intersect a distribution of lines?

$$\mathcal{N}(x_0|\boldsymbol{g}x, r^2) \cdot \mathcal{N}(x_0|\boldsymbol{W}t + \boldsymbol{\mu}, s^2)$$

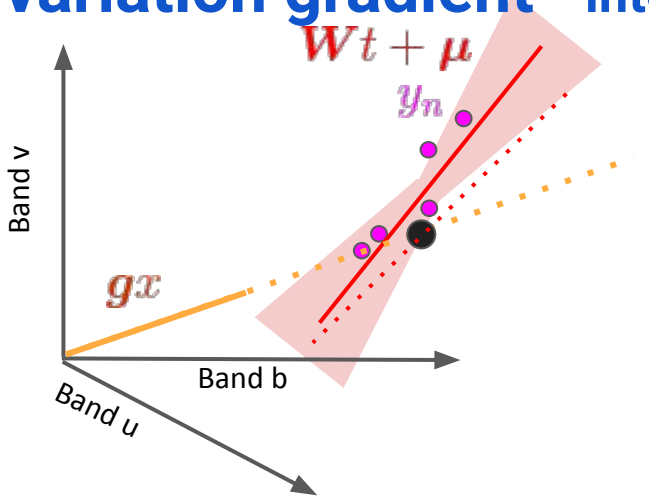
Probabilistic flux variation gradient - intersection



What does it mean to intersect a distribution of lines?

$$\lim_{r \to 0} \mathcal{N}(x_0 | \boldsymbol{g}x, r^2) \cdot \mathcal{N}(x_0 | \boldsymbol{W}t + \boldsymbol{\mu}, s^2) = \mathcal{N}(\boldsymbol{g}x | \boldsymbol{W}t + \boldsymbol{\mu}, s^2)$$

Probabilistic flux variation gradient - intersection

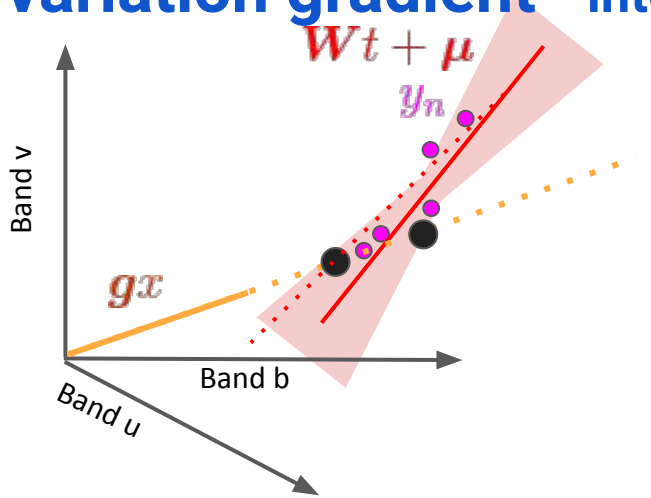


What does it mean to intersect a distribution of lines?

$$\text{account for all } \boldsymbol{W} \sim p(\boldsymbol{W}|\mathcal{D}),$$

$$\lim_{r \to 0} \mathcal{N}(x_0|\boldsymbol{g}x, r^2) \cdot \mathcal{N}(x_0|\boldsymbol{W}t + \boldsymbol{\mu}, s^2) = \mathcal{N}(\boldsymbol{g}x|\boldsymbol{W}t + \boldsymbol{\mu}, s^2)$$

Probabilistic flux variation gradient - intersection



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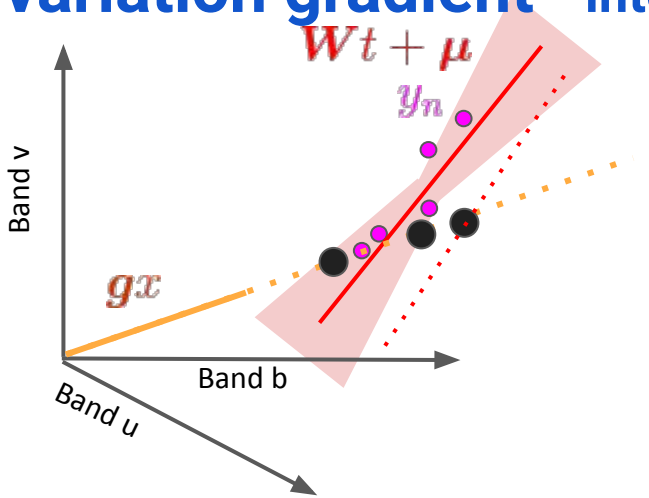
Probabilistic flux variation gradient - intersection



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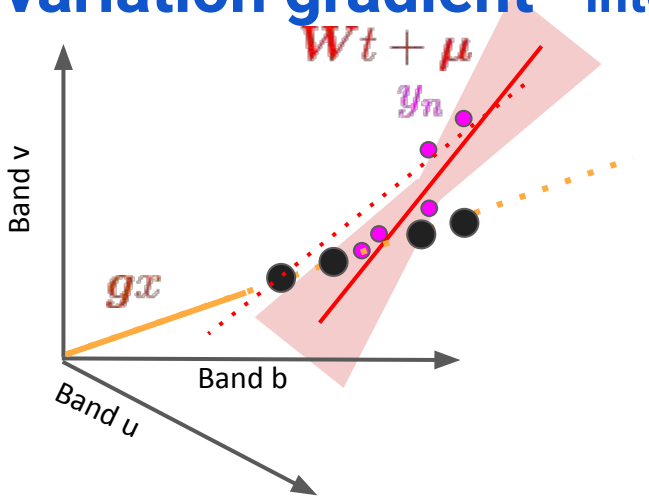
Probabilistic flux variation gradient - intersection



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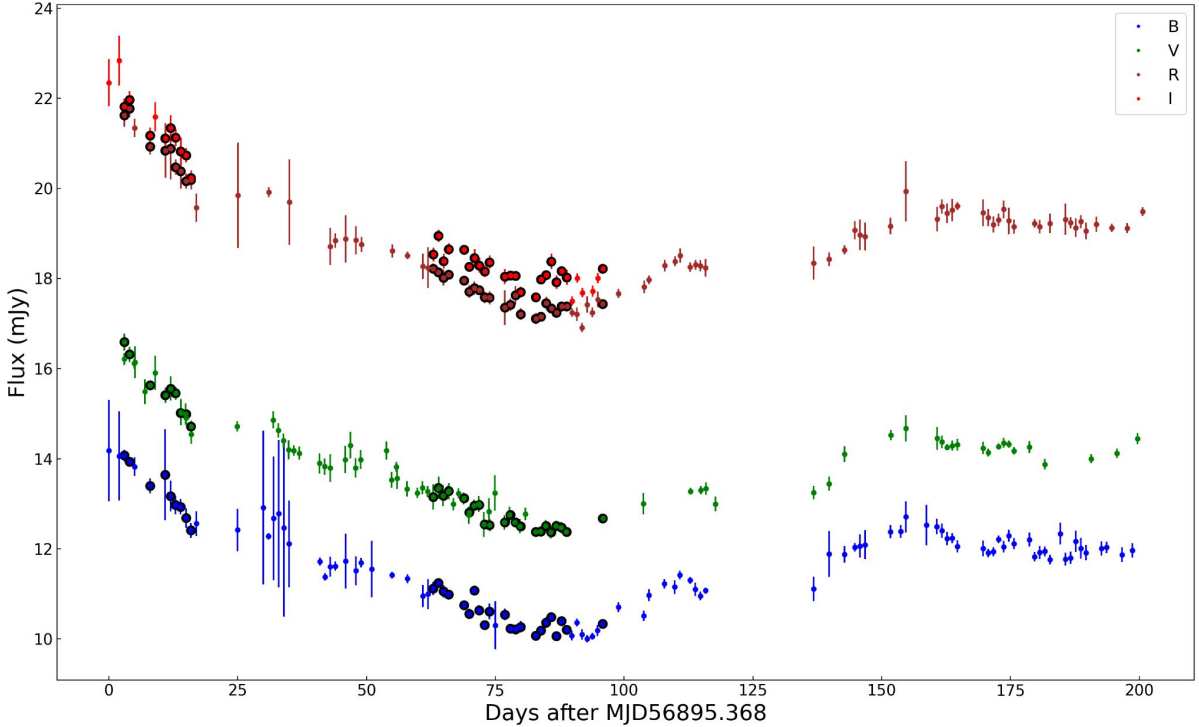
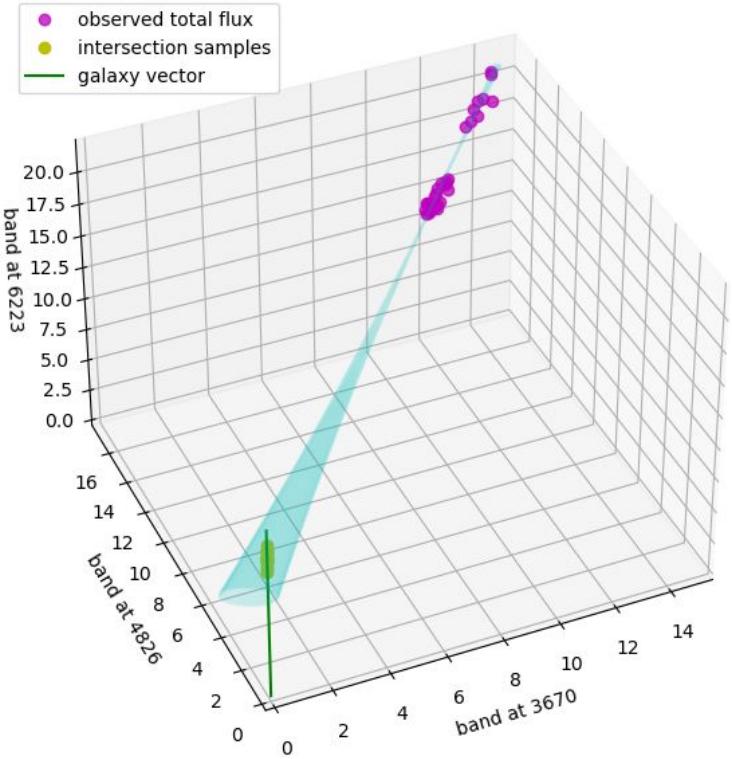
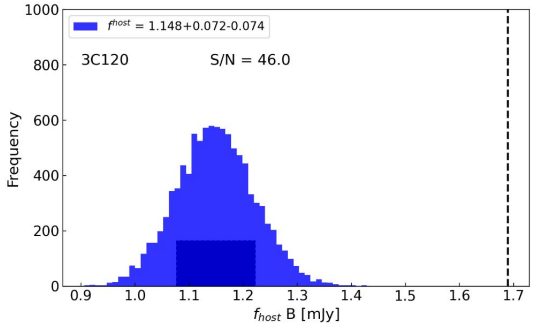
Application on 3C120

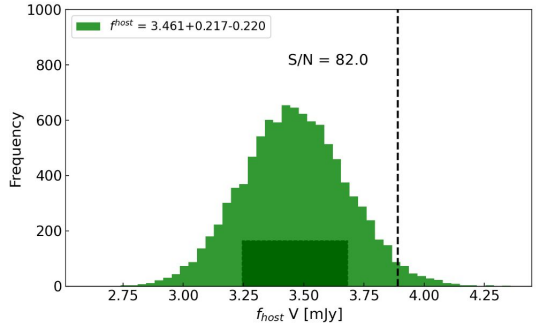


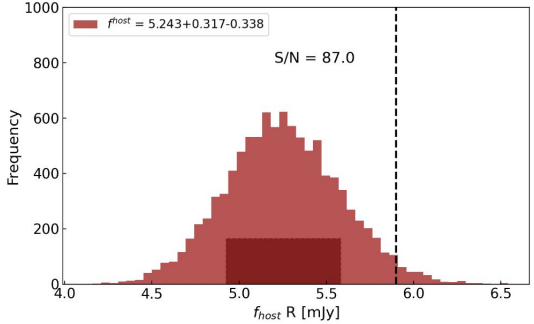
3C120 is a nearby Fanaroff-Riley Class I radio galaxy at redshift 0.033
We adopt a host-galaxy color given by a bulge S0 template (Kinney 1996)

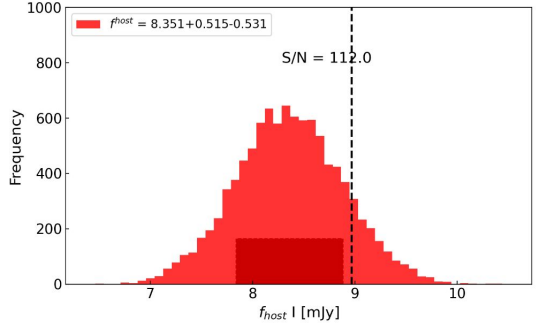
Application on 3C120











Conclusions

- We presented a probabilistic version of the flux variation gradient
- New method retains simplicity but accounts for uncertainty in observations and estimates
- Undestood as the intersection of the galaxy line with a distribution of total flux lines
- Interested?

  Disentangling the optical AGN and Host-galaxy luminosity with a probabilistic Flux Variation Gradient

  https://arxiv.org/abs/2109.03619

Application on 3C120